Operator mixing and the AdS/CFT Correspondence

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Introduction

- AdS/CFT context: Operator Mixing crucial when we have to compare observables on the two sides of the correspondence
- Planar $\mathcal{N}=4$ SYM is governed by the superconformal properties of the theory:
  - diagonalization of the Dilatation Operator:

$$\{\mathcal{O}_i\} \qquad \langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \mathbb{D}_{ij}$$

$$\mathbb{D}\hat{\mathcal{O}}_i = \Delta_i\hat{\mathcal{O}}_i \qquad \left\langle \hat{\mathcal{O}}_i(x)\hat{\mathcal{O}}_j(0)\right\rangle = \frac{\delta_{ij}}{x^{2\Delta_i}}$$

  - $\mathcal{N}=4$ SYM superalgebra highest weight state:

$$[S,\mathcal{O}(x=0)] = 0 \qquad [Q,\mathcal{O}(x=0)] \neq 0$$

Outline of the talk

- Remarks about the AdS/CFT Correspondence
- BMN/PP-wave subsector
- HWS in string theory
- Computation of the quantum corrections to the SYM supercharges
- Computation of the mixing terms for the two-impurity multiplet Highest Weight State (HWS)
- Role of the mixing terms in the computation of the structure constants between two half-BPS and one non-BPS operator.

Remarks about AdS/CFT

Type IIB String Theory on $AdS_5 x S^5$ ⟷ $\mathcal{N}=4$ SYM on the boundary of AdS_5

Symmetry Group PSU(2,2|4)

$\mathbf{SO(4,2)} \times \mathbf{SO(6)}$

- $SO(4,2)$: Isometry group of AdS_5 / Conformal symmetry group
- $SO(6)$: Isometry group of S^5 / R-symmetry group

Parameters

- N - F_5 flux units through S^5
- χ_0 - Axion vev
- R - AdS_5 and S^5 radius $\frac{R^2}{\alpha'} = \sqrt{4\pi N g_s}$

- N - rank of the gauge group $SU(N)$
- g - YM exp. $\lambda = g^2 N$
- θ - Instantonic angle

$$\frac{\theta}{2\pi} + i\frac{4\pi}{g^2} = \chi_0 + \frac{i}{g_s}$$

allows to identify

✓ $\frac{R^2}{\alpha'} = \sqrt{\lambda}$

✓ $g_s = \frac{g^2}{4\pi} = \frac{\lambda}{4\pi N}$

⟹

> 't Hooft limit ⟷ classical string theory

> strong c. limit ⟷ supergravity

Strong/Weak Duality

- Observables in the string side are associated to observables in the gauge theory side. They must coincide at EACH ORDER in the coupling constant:

$$E(\lambda) \quad \longleftrightarrow \quad \Delta(\lambda)$$

- BUT they are investigable in their perturbative regimes:

  ✓ $\Delta(\lambda) = \Delta_0 + \lambda\Delta_1 + \lambda^2\Delta_2 + \mathcal{O}(\lambda^3) \qquad \lambda \to 0$

  ✓ $E(\lambda) = \sqrt{\lambda}E_0 + E_1 + E_2/\sqrt{\lambda} + \mathcal{O}(\lambda^{-1}) \qquad \lambda \to \infty$

- To overcome this weak/strong coupling nature of AdS/CFT

  ✓ Computation of protected quantities

  ✓ Integrability

  ✓ Focus on more tractable subsectors: PP-wave/BMN

PP-wave string theory

Type IIB string theory on $AdS_5 x S^5$

$$ds^2 = R^2 \left[-\cosh^2 r dt^2 + dr^2 + \sinh^2 r d\Omega_3^2 + \cos^2 \theta d\psi^2 + d\theta^2 + \sin^2 \theta d\Omega'^2_3 \right]$$

$$F_5 = \frac{1}{R}(dV_{\mathrm{AdS}_5} + dV_{\mathrm{S}^5}) \qquad \phi = \text{const.}$$

- Introduce Light-Cone coordinate set:

$$x^+ = \frac{t+\psi}{2\mu} \qquad x^- = \mu R \frac{t-\psi}{2} \qquad \hat{r} = Rr \qquad y = R\theta$$

- Perform the $R \to \infty$ limit keeping $x^\pm$, $\hat{r}$, y fixed

$$g_{-+} = g_{+-} = 2$$

$$g_{IJ} = \delta_{IJ}, \quad I, J = 1, \ldots, 8$$

$$g_{++} = -\mu^2 \sum_{I=1}^{8} x_I x^I$$

$$F_{+1234} = F_{+5678} = 2\mu$$

SHORT STRINGS ROTATING FAST AROUND S^5

SYMMETRY GROUP $SO(4) \times SO(4)$

PP-wave string theory

- Full interacting string theory
- Free two-dimensional action: quantizable in term of eight towers of bosonic and fermionic oscillators

$$(a_n^i, a_n^{i'}) \quad (b_n^{\alpha_1\alpha_2}, b_{n\,\dot\alpha_1\dot\alpha_2}) \qquad n \in \mathbb{N}$$

- Physical spectum: Fock space of states $|s\rangle$ built upon a vacuum $|\alpha \equiv \alpha' p^+\rangle$

$$H = \frac{1}{\mu}\sum_{n=-\infty}^{+\infty} \frac{\omega_n}{\alpha}\left[a_n^\dagger a_n + b_n^\dagger b_n\right]$$

- Maximally supersymmetric background
  - eight kinematical charges $Q^+, \quad \bar{Q}^+$
  - eight dynamical charges $Q^-, \quad \bar{Q}^-$

Supercharges in the PP-wave background

$$Q^+ = \sqrt{2|\alpha|}\left[e(\alpha)\mathbb{P}^- b_0 + \mathbb{P}^+ b_0^\dagger\right]\ ,\quad \bar{Q}^+ = \sqrt{2|\alpha|}\left[\mathbb{P}^- b_0^\dagger + e(\alpha)\mathbb{P}^+ b_0\right]$$

$$\begin{aligned}
Q^- &= \sqrt{\frac{1}{2}}\gamma\left[a_0\mathbb{P}^+ b_0^\dagger + a_0^\dagger \mathbb{P}^- b_0\right] + \\
&+ \frac{1}{\sqrt{|\alpha|}}\sum_{n=1}^{\infty}\sqrt{n}\gamma\left[a_n^\dagger P_n b_{-n} + a_n P_n^{-1} b_n^\dagger + \mathrm{i}a_{-n}^\dagger P_n b_n - \mathrm{i}a_{-n} P_n^{-1} b_{-n}^\dagger\right] \\
\bar{Q}^- &= \sqrt{\frac{1}{2}}\gamma\left[a_0\mathbb{P}^- b_0^\dagger + a_0^\dagger \mathbb{P}^+ b_0\right] + \\
&+ \frac{1}{\sqrt{|\alpha|}}\sum_{n=1}^{\infty}\sqrt{n}\gamma\left[a_n^\dagger P_n^{-1} b_n + a_n P_n b_{-n}^\dagger + \mathrm{i}a_{-n}^\dagger P_n^{-1} b_{-n} - \mathrm{i}a_{-n} P_n b_n^\dagger\right]
\end{aligned}$$

The dual BMN limit

$\mathcal{N}=4$ SYM field content

- ✓ 4+4 Weyl fermions $\psi^\alpha_A, \quad \bar{\psi}^A_{\dot\alpha}$
- ✓ 6 scalars Φ^{AB} (or $Z_i, \bar{Z}_i$) $\quad A, B = 1, \ldots, 4$
- ✓ Gauge connection A_μ

In terms of $\mathcal{N}=2$ multiplets the R-symmetry group reads

$$SU(2)_V \times SU(2)_H \times U(1)_J$$

- Identify
  - $SU(2)_V \times SU(2)_H \longleftrightarrow SO(4)_{fl}$
  - $U(1)_J \longleftrightarrow x^-$ translations ($J \to \infty$)
- Rewrite the x^+ and x^- translation generators in terms of $\partial/\partial\psi, \partial/\partial t$. The Penrose limit translate to:

$$\Delta \to \infty \quad J \to \infty \quad N \to \infty$$

$$\frac{J}{\sqrt{N}}, \; \Delta - J, \; g^2 \quad \text{fixed}$$

Hamiltonian H/μ

perturbative expansion ⟹ $\lambda' \equiv \frac{g^2 N}{J^2} = \frac{1}{\mu\alpha}$

PP-wave/BMN dictionary

Gauge invariant operators VS string states transforming in the same representation under the $SO(4)\times SO(4)$ symmetry group

- Vacuum $\frac{H}{\mu}|\alpha\rangle = 0 \longrightarrow \Delta = J \Longrightarrow |\alpha\rangle \leftrightarrow \mathrm{Tr}\left[Z^J\right]$
- Half-BPS states $\Delta - J = N$

$$\mathrm{Tr}\left[Z^J\phi^{AB}\right] \longleftrightarrow a_0^i|\alpha\rangle$$

$$\sum_{p=0}^{J}\mathrm{Tr}\left[\Phi^{AB}Z^p\Phi^{CD}Z^{J-p}\right] \longleftrightarrow a_0^i a_0^j|\alpha\rangle$$

- Two-impurity string states

$$\sum_{p=0}^{J}\cos\frac{\pi n(2p+3)}{J+3}\left[\Phi_{AB}Z^p\Phi^{AB}Z^{J-p}\right] \longleftrightarrow \sum_{i'=1}^{4}\left(a^{\dagger}{}_n^{i'}a^{\dagger}{}_n^{i'} + a^{\dagger}{}_{-n}^{i'}a^{\dagger}{}_{-n}^{i'}\right)|\alpha\rangle$$

PP-wave/BMN string and perturbative N=4SYM

- BMN is a complementary regime wrt $\lambda \to 0$
- How do we use the double scaling limit?
- Perturbative expansion in $\frac{1}{\mu\alpha} \equiv \lambda' = \frac{\lambda}{J^2}$
- We have relied on this to compare the mixing coeffcients we have obtained in the two theories

CHANGE PERSPECTIVE

The multiplet is a representation of the superconformal algebra of the full interacting theory

$$[S, O(x=0)] = 0 \qquad [Q, O(x=0)] \neq 0$$

- Identify the supersymmetry generators in the two descriptions

$$Q_{\alpha,A=1,2} \leftrightarrow \mathbb{P}^+ Q^+ \quad Q_{\alpha,A=3,4} \leftrightarrow \mathbb{P}^+ Q^- \quad \bar{Q}^{\dot{\alpha},A=1,2} \leftrightarrow \mathbb{P}^- \bar{Q}^- \quad \bar{Q}^{\dot{\alpha},A=3,4} \leftrightarrow \mathbb{P}^- \bar{Q}^+$$

$$S_\alpha^{A=1,2} \leftrightarrow \mathbb{P}^+ \bar{Q}^+ \quad S_\alpha^{A=3,4} \leftrightarrow \mathbb{P}^+ \bar{Q}^- \quad \bar{S}^{\dot{\alpha}}_{A=1,2} \leftrightarrow \mathbb{P}^- Q^- \quad \bar{S}^{\dot{\alpha}}_{A=3,4} \leftrightarrow \mathbb{P}^- Q^+$$

- In the string sector the two impurity HWS is

$$\mathbb{P}^+ \bar{Q}^+ |hws\rangle = \mathbb{P}^+ \bar{Q}^- |hws\rangle = \mathbb{P}^- Q^- |hws\rangle = \mathbb{P}^- Q^+ |hws\rangle = 0$$

String Highest Weight State

$$|n\rangle = \frac{1}{4(1+U_n^2)}\left[{a^\dagger}_n^{i'}{a^\dagger}_n^{i'} + {a^\dagger}_{-n}^{i'}{a^\dagger}_{-n}^{i'} + 2U_n b^\dagger_{-n}\,\Pi\, b^\dagger_n - U_n^2\left({a^\dagger}_n^{i}{a^\dagger}_n^{i} + {a^\dagger}_{-n}^{i}{a^\dagger}_{-n}^{i}\right)\right]|\alpha\rangle$$

- Valid for finite $\mu\alpha$: full interacting string theory
- Mixing between different type of impurities
- Leading order for $\mu\alpha \to \infty$ coincide with the old dictionary
- Leading and subleading corrections: fermion impurities and derivative impurities respectively
- Straightforward to build the whole multiplet

Operator Mixing in $\mathcal{N}=4$ SYM

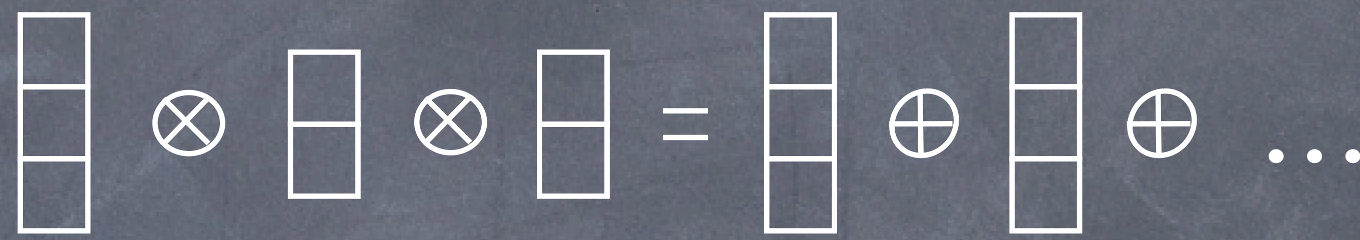
Free Theory: $\delta_S \Phi^{AB}(0) = 0 \longrightarrow S \sum_{p=0}^{J} \cos\frac{\pi n(2p+3)}{J+3} \mathrm{Tr}\left[\Phi_{AB} Z^p \Phi^{AB} Z^{J-p}\right] = 0$

Interacting theory: supersymmetry and superconformal charges get quantum corrections

order-g $\longrightarrow$ $S\Phi\Phi \propto g\psi$, $S\Phi\psi \propto g D_\mu \Phi$

$$\mathcal{O}_n^J \propto \sum_{i=1}^{3}\sum_{p=0}^{J} \cos\frac{\pi n(2p+3)}{J+3} \mathrm{Tr}\left[Z_i Z^p \bar{Z}_i Z^{J-p}\right]$$
$$+ g\mathcal{C}_1 \sum_{p=0}^{J-1} \sin\frac{\pi n(2p+4)}{J+3} \left(\mathrm{Tr}\left[\psi^{1\alpha} Z^p \psi^2_\alpha Z^{J-1-p}\right] - \mathrm{Tr}\left[\bar{\psi}_{3\dot{\alpha}} Z^p \bar{\psi}_4^{\dot{\alpha}} Z^{J-1-p}\right]\right)$$
$$+ g^2\mathcal{C}_2 \sum_{p=0}^{J-2} \cos\frac{\pi n(2p+5)}{J+3} \mathrm{Tr}\left[D_\mu Z Z^p D^\mu Z Z^{J-p-2}\right] + \mathcal{O}(g^3)$$

$$\bar{S}^{\dot{\alpha}}_{A}\Phi_{BC}\Phi_{DE}(0) = -\mathrm{i}\,\frac{gN}{32\pi^2}\left(\epsilon_{ABC[D}\bar{\psi}_{E]}(0) - \epsilon_{ADE[B}\bar{\psi}^{\dot{\alpha}}_{C]}(0)\right)$$

✓ $SU(4)$ structure: 

✓ Numerical Coefficient: Ward Identities

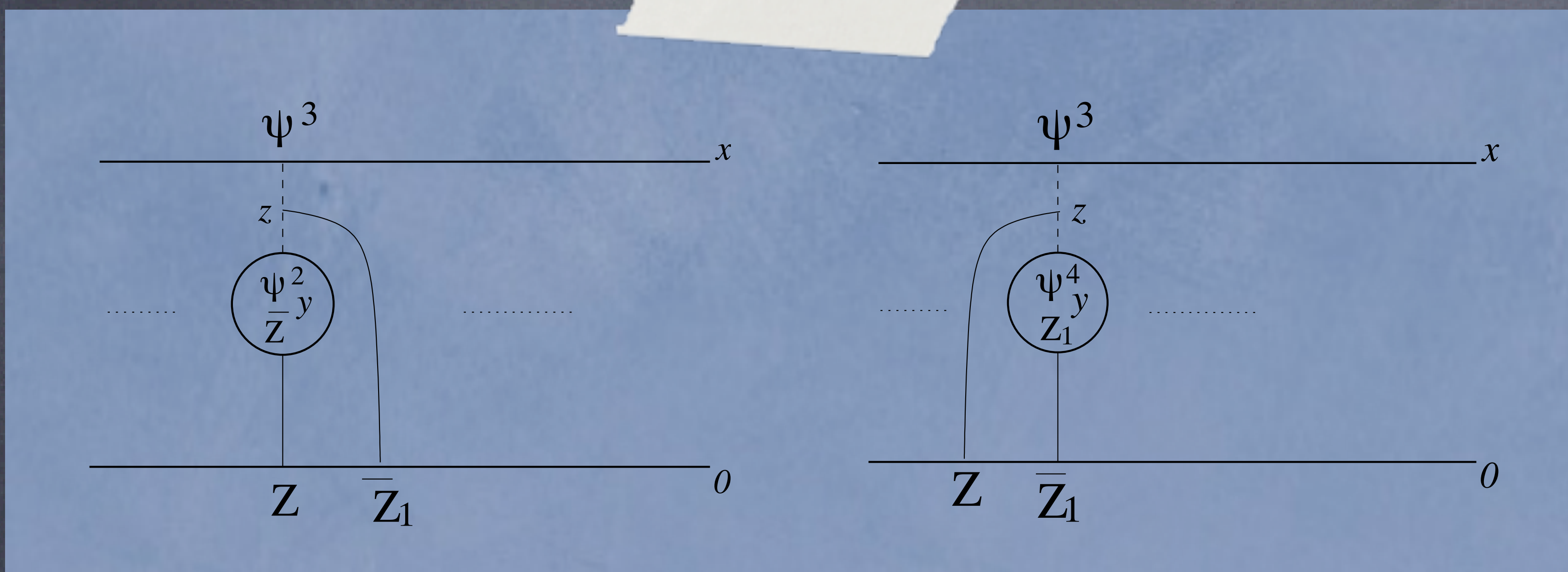
$$\partial^y_\mu\langle O_1(x)S^\mu(y)O(0)\rangle = -\mathrm{i}\delta^4(x-y)\langle\delta_S O_1(x)\,O(0)\rangle + \mathrm{i}\,\delta^4(y))\langle O_1(x)\,\delta_S O(0)\rangle$$

The superconformal current relevant for this computation is

$$\bar{S}^{\mu\dot{\alpha}}_{A} = 2x_\tau(\bar{\sigma}^\tau)^{\dot{\alpha}\alpha}\mathrm{Tr}\Big[(\sigma^{\rho\nu})_\alpha^{\ \beta}F_{\rho\nu}\sigma^\mu_{\beta\dot{\beta}}\bar{\psi}^{\dot{\beta}}_A + 2\sqrt{2}\sigma^\rho_{\alpha\dot{\alpha}}\bar{\sigma}^{\mu\,\dot{\alpha}\beta}D_\rho\Phi_{AB}\psi^B_\beta +$$
$$- 4\mathrm{i}g\sigma^\mu_{\alpha\dot{\beta}}[\Phi_{AC},\Phi^{CB}]\bar{\psi}^{\dot{\beta}}_B\Big] + 8\sqrt{2}\bar{\sigma}^{\mu\,\dot{\alpha}\alpha}\mathrm{Tr}\big[\Phi_{AB}\psi^B_\alpha\big]$$

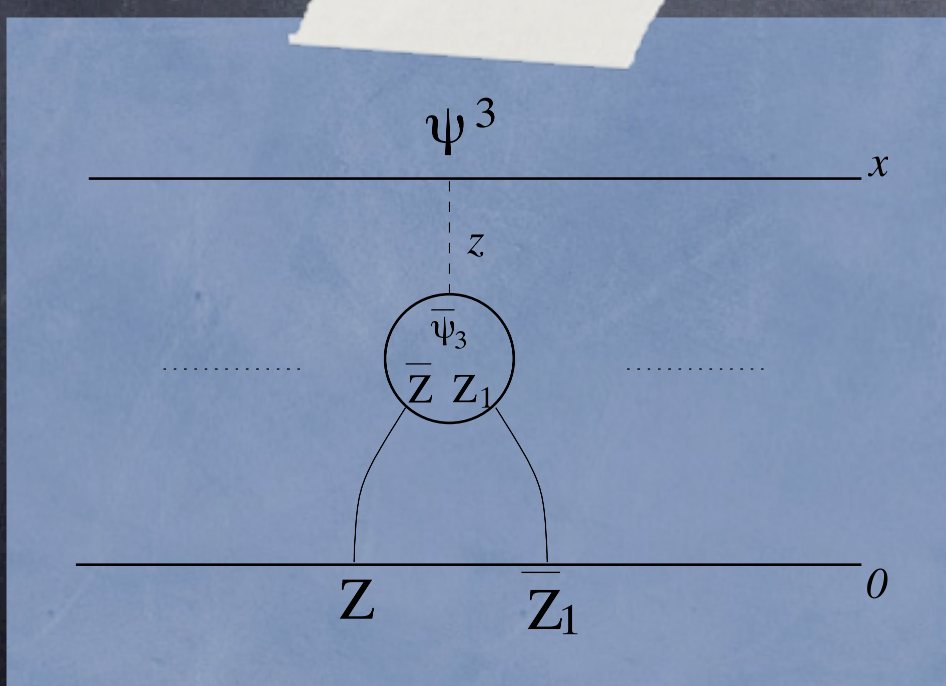
$$\langle \psi^3_\gamma(x)\, \bar{S}^{\mu\dot{\alpha}}_1(y)\, Z\bar{Z}_1(0)\rangle$$

- Second and last terms of the current



→ The contributions from the two terms cancel each others

- Third term of the current



→

$$G^\mu_3 = \frac{gN}{16\pi^2}\bar{\sigma}^{\tau\dot{\alpha}\alpha}\,\partial^y_\tau\Delta(y)\,\sigma^\mu_{\alpha\dot{\beta}}\,\epsilon^{\dot{\beta}\dot{\gamma}}\,\sigma^\nu_{\gamma\dot{\gamma}}\,\partial^x_\nu\Delta(x-y)$$

$$\partial^y_\mu G^\mu_3 = -\frac{igN}{16\pi^2}(\delta^{(4)}(y)\epsilon^{\dot{\alpha}\dot{\gamma}}\,\sigma^\nu_{\gamma\dot{\gamma}}\,\partial^x_\nu\Delta(x)+\ldots)$$

↓

$$\bar{S}^{\dot{\alpha}}_1 Z\bar{Z}_1 = \frac{-igN}{8\pi^2}\bar{\psi}^{\dot{\alpha}}_3$$

Similarly we can compute:

- $\bar{S}_1^{\dot{\alpha}} \bar{\psi}_{B\dot{\beta}} = 4\sqrt{2}\mathrm{i}\Phi_{AB}(0)\delta_{\dot{\beta}}^{\dot{\alpha}}$
- $\bar{S}_1^{\dot{\alpha}} \not{D}_{\alpha\dot{\beta}} Z = 2\sqrt{2}\psi_\alpha^2 \delta_{\dot{\beta}}^{\dot{\alpha}}$
- $\bar{S}_1^{\dot{\alpha}}(\psi_\alpha^1 Z) = -\frac{gN}{8\pi^2}\bar{\sigma}^{\mu\,\dot{\alpha}}{}_{\alpha} D_\mu Z$

$$\mathcal{O}_n^J = \sqrt{\frac{N_0^{-J-2}}{(J+3)}}\mathcal{Z}\sum_{i=1}^{3}\sum_{p=0}^{J}\cos\frac{\pi n(2p+3)}{J+3}\mathrm{Tr}\left[Z_i Z^p \bar{Z}_i Z^{J-p}\right]$$
$$+\frac{g\sqrt{N}}{4\pi}\sin\frac{\pi n}{J+3}\sqrt{\frac{N_0^{-J-1}}{(J+3)}}\sum_{p=0}^{J-1}\sin\frac{\pi n(2p+4)}{J+3}\left(\mathrm{Tr}\left[\psi^{1\alpha}Z^p\psi_\alpha^2 Z^{J-1-p}\right]-\mathrm{Tr}\left[\bar{\psi}_{3\dot{\alpha}}Z^p\bar{\psi}_4^{\dot{\alpha}}Z^{J-1-p}\right]\right)$$
$$+\frac{g^2 N}{16\pi^2}\sin^2\frac{\pi n}{J+3}\sqrt{\frac{N_0^{-J}}{(J+3)}}\sum_{p=0}^{J-2}\cos\frac{\pi n(2p+5)}{J+3}\mathrm{Tr}\left[D_\mu Z Z^p D^\mu Z Z^{J-p-2}\right]+\mathcal{O}(g^3)$$

<u>One can use the same algorithm to compute the whole multiplet</u>

Field Theory VS String Theory

We can compare the large $\mu\alpha = (\lambda')^{-1}$ limit of the string HWS with the large J expansion of $\mathcal{O}_n^J$:

$$|n\rangle \approx \frac{1}{4}\left[{a^\dagger}^{i'}_n {a^\dagger}^{i'}_n + {a^\dagger}^{i'}_{-n} {a^\dagger}^{i'}_{-n} - n\sqrt{\lambda'}\left({b^\dagger}_{-n}\mathbb{P}^+ {b^\dagger}_n - {b^\dagger}_{-n}\mathbb{P}^- {b^\dagger}_n\right)\right]|\alpha\rangle$$

$$\sum_{i=1}^{3}\sum_{p=0}^{J}\cos\frac{\pi n(2p+3)}{J+3}\mathrm{Tr}\left[Z_i Z^p \bar{Z}_i Z^{J-p}\right]$$

$$\sum_{p=0}^{J-1}\sin\frac{\pi n(2p+2)}{J+1}\left(\mathrm{Tr}\left[\psi^{1\alpha}Z^p\psi^2_\alpha Z^{J-p-1}\right] - \mathrm{Tr}\left[\bar{\psi}_{3\dot{\alpha}}Z^p\bar{\psi}^{\dot{\alpha}}_4 Z^{J-p-1}\right]\right)$$

- Fix the normalization: two-point functions get the canonical form
- Rewriting $\sqrt{\lambda'} = \frac{g\sqrt{N}}{J}$

$$(\mathcal{O}_{st})^J_n = \sqrt{\frac{N_0^{-J-2}}{J+3}}\sum_{p=0}^{J}\cos\frac{\pi n(2p+3)}{J+3}\mathrm{Tr}\left[Z_i Z^p \bar{Z}_i Z^{J-p}\right] +$$

$$+\frac{g\sqrt{N}n}{4J}\sqrt{\frac{N_0^{-J-1}}{J+1}}\sum_{p=0}^{J-1}\sin\frac{\pi n(2p+2)}{J+1}\left(\mathrm{Tr}\left[\psi^{1\alpha}Z^p\psi^2_\alpha Z^{J-p-1}\right] - \mathrm{Tr}\left[\bar{\psi}_{3\dot{\alpha}}Z^p\bar{\psi}^{\dot{\alpha}}_4 Z^{J-p-1}\right]\right)$$

Agrees with the large-J limit of the SYM HWS

Comments

- If J=0 Konishi operator: no mixing
- No overlap between leading and subleading terms at one loop. Test for the three-loop anomalous dimension in the full theory.
- Subleading terms do not alter the known results for the anomalous dimension
  - ✓ one loop at finite J
  - ✓ higher loops for large J
- Subleading terms are crucial to verify existing selection rules for OPE structure constants (correlators among two half-BPS and one non-BPS operator)

$U(1)_Y$ Bonus Symmetry I

- Originates from the $SL(2,\mathbb{R})$ symmetry of type IIB supergravity in 10 dimensions
- Not a symmetry of the $\mathcal{N}=4$ SYM Lagrangian. Exact symmetry of the equations of motion of the free theory and of the supergravity limit
- To assign a $U(1)_Y$ charge:
  - ✓ $Q^A_\alpha,\ \bar{S}^{\dot\alpha}_A$: $u_Y=+1$
  - ✓ $\bar{Q}^{\dot\alpha}_A,\ S^A_\alpha$: $u_Y=-1$

  → Bosonic Generators have charge zero

  - ✓ HWS: charge zero. Descendent: sum of the supersymmetry charges acting on HWS

$U(1)_Y$ Bonus Symmetry II

$U(1)_Y$ charge conservation selection rule

Three and four-point correlators of half-BPS operators

Three-point correlators with one non-BPS operator

Mixing terms are crucial to realize this constraint

$$\langle \mathcal{O}_0^{J_1,\bar{Z}_1\bar{Z}_2}(x_1)\,\mathcal{O}_0^{J_2,Z_1Z_2}(x_2)\,{}^{[4]}\bar{\mathcal{O}}_n^{J_3}(x_3)\rangle \qquad J_3 = J_1 + J_2 - 1$$

$$\mathcal{O}_0^{J_1,\bar{Z}_1\bar{Z}_2} = \sum_{k_1=0}^{J_1} \mathrm{Tr}\big[\bar{Z}_1 Z^{k_1}\bar{Z}_2 Z^{J_1-k_1}\big]$$

$$\mathcal{O}_0^{J_2,Z_1Z_2} = \sum_{k_2=0}^{J_2} \mathrm{Tr}\big[Z_1 Z^{k_2} Z_2 Z^{J_2-k_2}\big]$$

→ $u_Y=0$

$${}^{[4]}\mathcal{O}_n^{J_3} =(Q^3)^2(Q^4)^2\mathcal{O}_n^{J_3+2} \propto \sum_{p=0}^{J_3} \sin\frac{\pi n(2p+2)}{J_3+2}\mathrm{Tr}\big[\psi^{1\alpha}Z^p\psi^2_\alpha Z^{J_3-p}\big]+$$

$$-2\sqrt{2}g\sin\frac{\pi n}{J_3+2}\sum_{p=0}^{J_3+1}\cos\frac{\pi n(2p+1)}{J_3+2}\mathrm{Tr}\big[\Phi_{AB}Z^p\Phi^{AB}Z^{J_3-p+1}\big]+$$

→ $u_Y=4$

$$+\frac{gN}{8\sqrt{2}\pi^2}\sin\frac{\pi n}{J_3+2}\sum_{p=0}^{J_3-1}\cos\frac{\pi n(2p+3)}{J_3+2}\mathrm{Tr}\big[D_\mu Z Z^p D^\mu Z Z^{J_3-p-1}\big]+\mathcal{O}(g^2)$$

This correlator does not conserve the $U(1)_Y$ charge, then its coefficient must be zero

- First non-trivial contributions come at order g
- Conformal symmetry fixes the spacetime dependence to:

$$\Delta_{x_1x_2}\Delta_{x_1x_3}^{J_1+1}\Delta_{x_2x_3}^{J_2+1} \qquad \text{with} \qquad \Delta_{x_ix_j} \propto \frac{1}{(x_i - x_j)^2}$$

- Contributions to the structure constant:

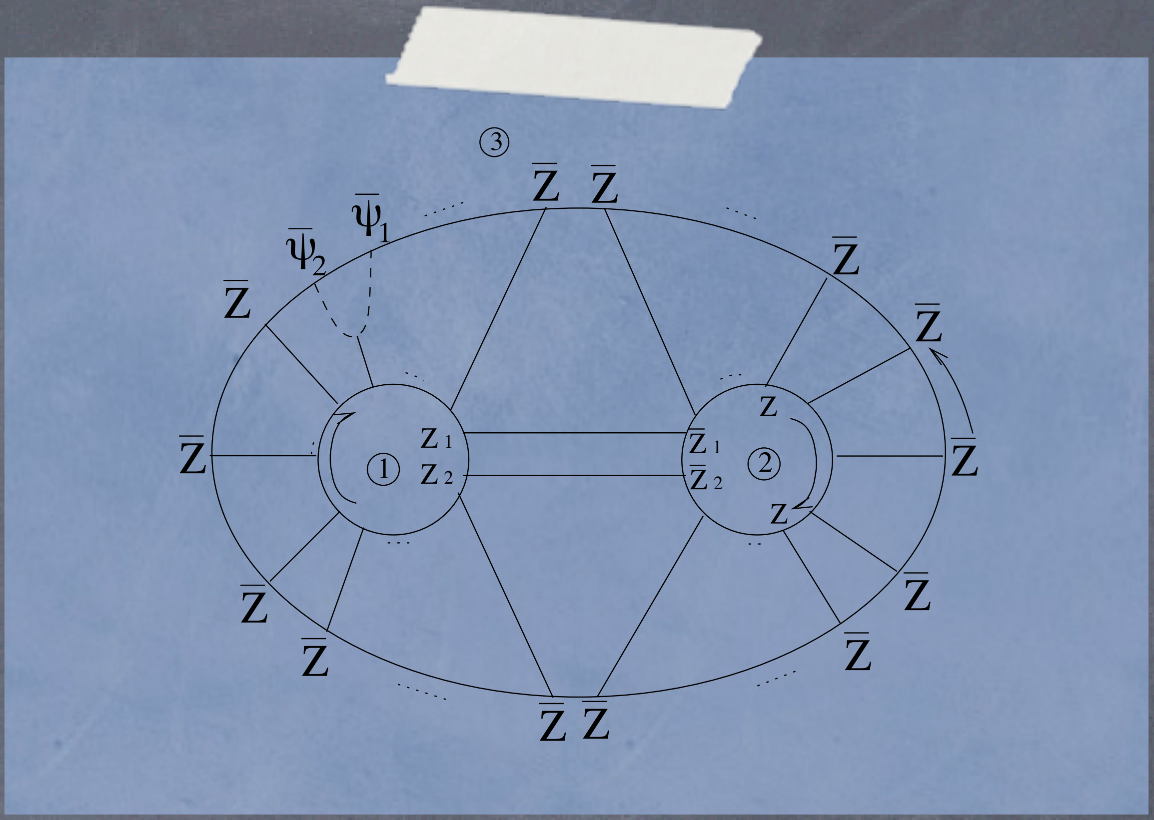
(1) contraction of the leading term of the non-BPS operator with the two half-BPS ones through a Yukava coupling $g\mathrm{Tr}[\psi\psi\Phi]$

(2) tree-level contraction of the subleading term of the non-BPS operator with fermionic impurities and the two half-BPS ones

(3) tree-level contraction of the subleading term of the non-BPS operator with derivative impurities and the two half-BPS ones

Contribution (1)

- Constraint: $k_1 = 0,\ k_2 = J_2$ and $k_1 = J_1,\ k_2 = 0$
- Contr with the Yukawa: $p = 0$ and $p = J_3$. Different sign compensated by the antisymmetric phase factor
- The Yukawa can be contracted with both the half-BPS operators



$$A_3^{(L)} = -\mathrm{i}\sqrt{2}\frac{gN^{J_3+3}}{2^{J_3+1}}\sin\frac{2\pi n}{J_3+2}\Delta^2_{x_1x_2}\Delta^{J_1-1}_{x_1x_3}\Delta^{J_2-1}_{x_2x_3}\times$$

$$\left(J_1\Delta_{x_2x_3}\int d^4z\Delta_{x_1z}\partial^{(x_3)}_\mu\Delta_{x_3z}\partial^{\mu(x_3)}\Delta_{x_3z} + J_1 \leftrightarrow J_2\right)$$

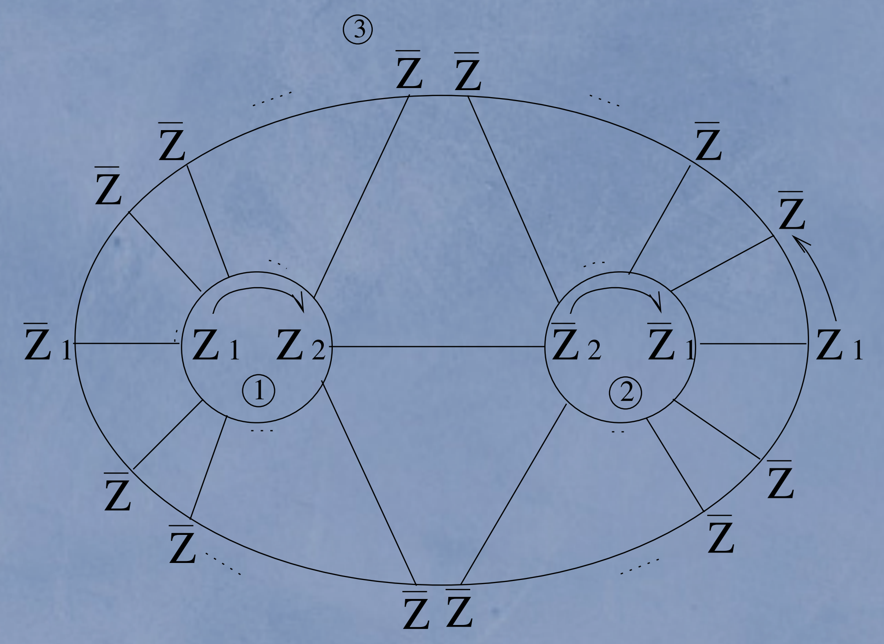
$$\rightarrow -\frac{\mathrm{i}}{2}\Delta^2_{x_1x_3}$$

$$A_3^{(L)} = -\frac{gN^{J_3+3}}{2^{J_3+1}\sqrt{2}}\sin\frac{2\pi n}{J_3+2}\Delta^2_{x_1x_2}\Delta^{J_1}_{x_1x_3}\Delta^{J_2}_{x_2x_3}\left(J_1\Delta_{x_1x_3} + J_2\Delta_{x_2x_3}\right)$$

DOES NOT TAKE THE FORM DICTATED BY CONFORMAL INVARIANCE!

Contribution (2)

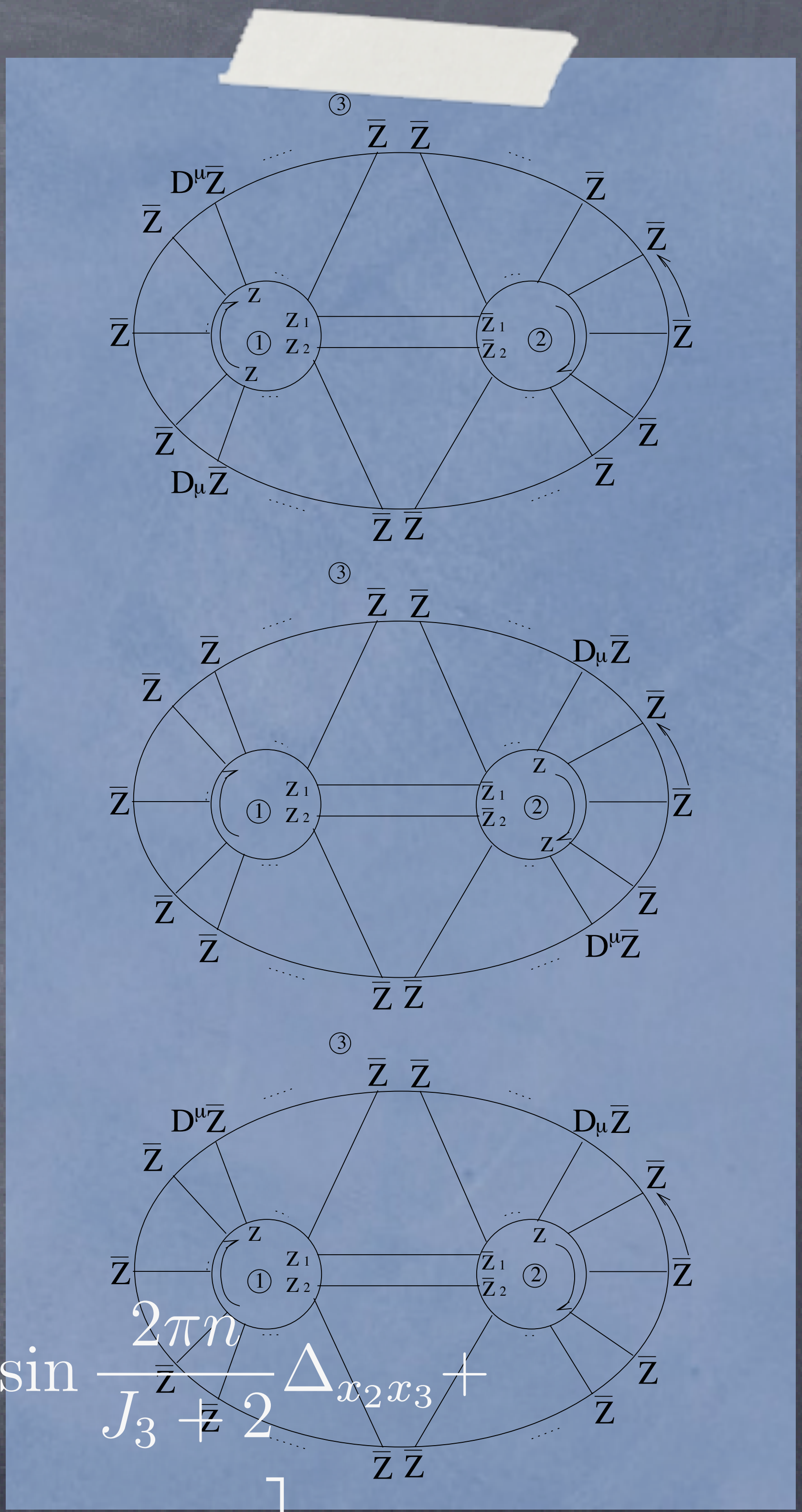
- Focusing on the term $\mathrm{Tr}[Z_1\ldots\bar{Z}_1\ldots]$: planar contractions for $p = k_1 - k_2 + J_2$
- Term $\mathrm{Tr}[Z_1\ldots\bar{Z}_1\ldots]$ double this result: planar contractions for $p = k_2 - k_1 + J_1$, mapped to the previous case by $p \longrightarrow J_3 - p + 1$ (phase factor symmetric)



Summing over the phases:

$$A_3^{(SL_f)} = \frac{gN^{J_3+3}}{2^{J_3+2}\sqrt{2}} \sin^{-1}\frac{\pi n}{J_3+2}\left(\cos\frac{\pi n}{J_3+2} - \cos\frac{\pi n(2J_1+1)}{J_3+2}\right)\Delta_{x_1x_2}\Delta_{x_1x_3}^{J_1+1}\Delta_{x_2x_3}^{J_2+1}$$

Contribution (3)



- Planarity constraints $k_1 = 0,\ k_2 = J_2$ and $k_1 = J_1,\ k_2 = 0$
- Multiplicity given by the values of p depends on the way we contract the derivative impurities
- First case: for any $p \in [0, J_1 - 2]$, multiplicity $J_1 - p - 1$
- Second case: just exchange $x_1 \leftrightarrow x_2$ and $J_1 \leftrightarrow J_2$
- Third case: introduce k counting the background fields following the impurity which are contracted with x_1. For any $k \in [0, J_1 - 1]$, we have $p \in [k, J_2 + k - 1]$
- In any case exchanging the role of the impurities doubles the result.

Summing over these diagrams we get

$$A_3^{(SL_d)} = \frac{gN^{J_3+3}}{2^{J_3+2}\sqrt{2}} \Delta^2_{x_1x_2} \Delta^{J_1}_{x_1x_3} \Delta^{J_2}_{x_2x_3} \left[2J_1 \sin\frac{2\pi n}{J_3+2} \Delta_{x_1x_3} + 2J_2 \sin\frac{2\pi n}{J_3+2} \Delta_{x_2x_3} + \right.$$
$$\left. - \sin^{-1}\frac{\pi n}{J_3+2} \left(\cos\frac{\pi n}{J_3+2} - \cos\frac{\pi n(2J_1+1)}{J_3+2} \right) \Delta^{-1}_{x_1x_2} \Delta_{x_1x_3} \Delta_{x_2x_3} \right]$$

CANCELS PRECISELY THE TWO PREVIOUS CONTRIBUTIONS

Summary and Comments

- We introduced a novel approach to the computation of the quantum correctios to the superconformal charges in the full PSU(2,2|4)-invariant theory
- The quantum corrected charges allow to solve the mixing for the gauge invariant operators with two impurities and dual string states in the BMN limit
- Mixing with novel structures with both fermionic and derivative impurities appearing at order g and g^2
- The new terms do not alter the known results for the anomalous dimension of the operators we studied

- Mixing terms are crucial to fulfill selection rules for the structure constants between two half-BPS and one non-BPS operators
- Computed the same structure constant in the PP-wave strings theory: the correlator I presented is actually non-zero in the dual theory
- $U(1)_Y$ selection rule is exact in the couplings: what is the origin of the mismatch?
- Study the higher order corrections to the superconformal charges. What about the divergencies?
- Yangian symmetry of three-point functions?