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On the probability of major-axis precession in triaxial ellipsoidal potentials

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ABSTRACT

Orbits in triaxial ellipsoidal potentials precess about either the major or minor axis of the ellipsoid. In standard perturbation theory, it can be shown that a circular orbit will precess about the minor axis if its angular momentum vector lies in a region bounded by two great circles that pass through the intermediate axis, and which are inclined with minimum separation i_T from the minor axis, where $i_T = \arctan[(B^2 - C^2)/(A^2 - B^2)]^{1/2}$, and A, B and C are the axis ratios, $A \geq B \geq C$. We test the accuracy of this formula by performing orbit integrations to determine i_S, the simulated turnover angle corresponding to i_T.

We reach two principal conclusions: (i) i_S is usually greater than i_T, by as much as 12°, even for moderate triaxialities $A/1.2 < B < C/0.8$. This reduces the expected frequency of polar rings. (ii) i_S is not a single, well-defined number, but can vary by a few degrees, depending upon the initial phase of the orbit. This means that there is a reasonable probability for capture of gas on to orbits that precess about both axes. Interactions can then lead to substantial loss of angular momentum and subsequent infall to the Galactic Centre.

Key words: celestial mechanics, stellar dynamics – galaxies: formation – galaxies: kinematics and dynamics – galaxies: peculiar.

1 INTRODUCTION

Polar ring galaxies are systems in which a ring of gas (and/or young stars) is seen to orbit about the major axis of an early-type galaxy. In many cases, it has been established that the host galaxy is rotating at right angles to the ring, and this is sometimes taken to be part of the definition of a polar ring system. There are just six confirmed polar rings, but 27 good candidates and many more possibles – see Whitmore et al. (1990) for a review.

The host galaxy in polar ring systems often appears to be S0, but this leads to theoretical problems: in an oblate, axisymmetric system, all orbits will precess about the minor axis at a rate that is a function of radius (typically the period is proportional to radius). This differential precession will cause the polar ring to fragment. This can be overcome if a small degree of triaxiality is assumed, as orbits whose angular momentum is sufficiently close to the major axis will then precess around the major rather than the minor axis. In general, accreted discs of gas will have an arbitrary orientation. In this case, differential precession about a symmetry axis, coupled with dissipation, will lead to collapse into the plane perpendicular to that axis (see, for example, Durisen et al. 1983).

The regions of phase space that lead to precession about the major or minor axes can be investigated using perturbation theory in Hamiltonian mechanics. It is assumed that the unperturbed potential is spherically symmetric, and that the orbits are circular. The Hamilton is then expanded in spherical harmonics, and the time-averaged perturbing potential calculated (see, for example, Steiman-Cameron & Durisen 1984). This results in an expression of the form

$$\langle \Phi_1 \rangle \propto C_{20}(3\sin^2 i - 2) + 6C_{22}\cos 2\Omega \sin^2 i, \tag{1}$$

where Ω and i are the node and inclination of the orbit, as illustrated in Fig. 1, and C_{20} and C_{22} are constants. Orbits will precess along lines $\langle \Phi_1 \rangle$ = constant, as shown in Fig. 2. If the angular momentum of the orbit, $\boldsymbol{J}$, lies within the shaded region, then it will precess about the major axis; otherwise it will precess about the minor axis. The dividing lines between the two regions are great circles which pass through the intermediate axis, and which are inclined at an angle

$$i_T = \arcsin\left(\frac{C_{20} + 2C_{22}}{C_{20} - 2C_{22}}\right)^{1/2} \tag{2}$$

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to the minor axis. Hence the probability that a randomly inclined disc will precess around the major axis is

$$f = 1 - \frac{2 i_{\mathrm{T}}}{\pi}.$$

This formula is extensively used in the theory of polar rings. It is generally accepted that dissipation will cause a disc of gas to settle into a plane perpendicular to the axis about which it precesses. Even a small triaxiality can lead to a reasonable probability for polar ring formation. For example, taking an ellipsoidal density distribution with semi-axes $a = 1.01$, $b = 1$ and $c = 0.8$, we obtain $i_{\mathrm{T}} \approx 77°$ and $f \approx 0.15$.

In this paper, we test the validity of the perturbation theory by calculating orbits in an ellipsoidal triaxial potential. We find that the measured transition angle, i_{S}, which divides precession about the major and minor axes, is usually larger than the theoretical value, i_{T}. Also, the boundary dividing the two regions of precession about the major and minor axes is not sharp. The precession axis depends upon the initial phase of the orbit, and i_{S} can vary by a few degrees. The integration method and results of our simulations are presented in Section 2. The conclusions are summarized and discussed in Section 3.

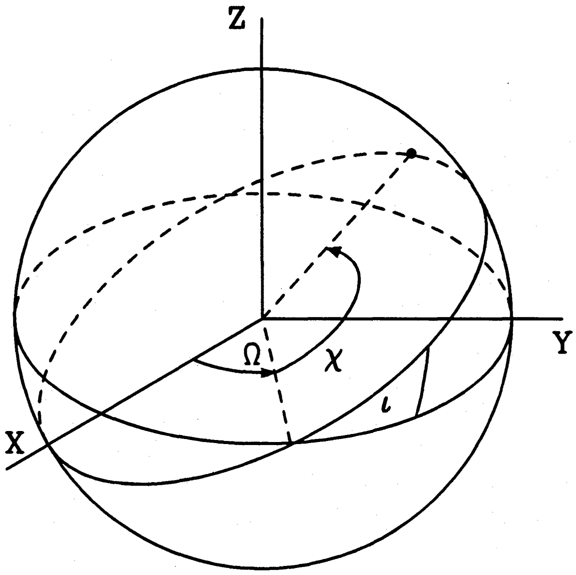

Figure 1. The orbital elements.

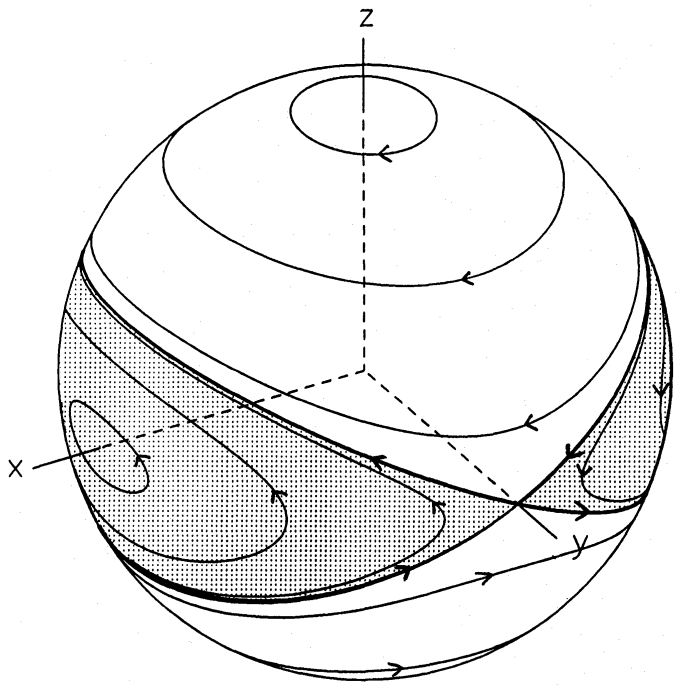

Figure 2. Precessional trajectories for the angular momentum vector of near-circular orbits in a triaxial potential (after Steiman-Cameron & Durisen 1982). The x-, y- and z-directions correspond to the major, intermediate and minor axes, respectively.

2 INTEGRATION METHOD AND RESULTS

2.1 The integration method

The numerical code used in this paper is that described by Pearce & Thomas (1991). It is a simple predictor-corrector method which integrates orbits to high accuracy, and we refer the reader to their paper for details.

We use a potential

$$\phi = \ln\left(\frac{x^2}{A^2} + \frac{y^2}{B^2} + \frac{z^2}{C^2}\right),$$

where $A \geq B \geq C$ and, without loss of generality, we can take $B = 1$. The time-averaged, first-order perturbation potential takes the form of equation (1), with

$$C_{20} = \frac{1}{A^2} + \frac{1}{B^2} - \frac{2}{C^2},$$

$$C_{22} = \frac{1}{2}\left(\frac{1}{B^2} - \frac{1}{A^2}\right),$$

which gives, on substitution in equation (2),

$$i_{\mathrm{T}} = \arcsin\left(\frac{1/C^2 - 1/B^2}{1/C^2 - 1/A^2}\right)^{1/2} = \arctan\left(\frac{B^2 - C^2}{A^2 - B^2}\right)^{1/2}.$$

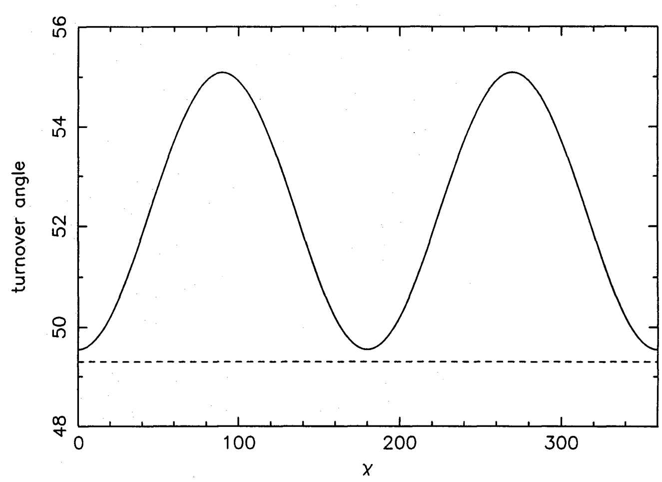

Figure 3. The measured turnover angle, i_{S}, as a function of initial phase for the case $A = 1.1$, $B = 1.0$ and $C = 0.9$. The theoretical value, i_{T}, is shown as a dashed line.

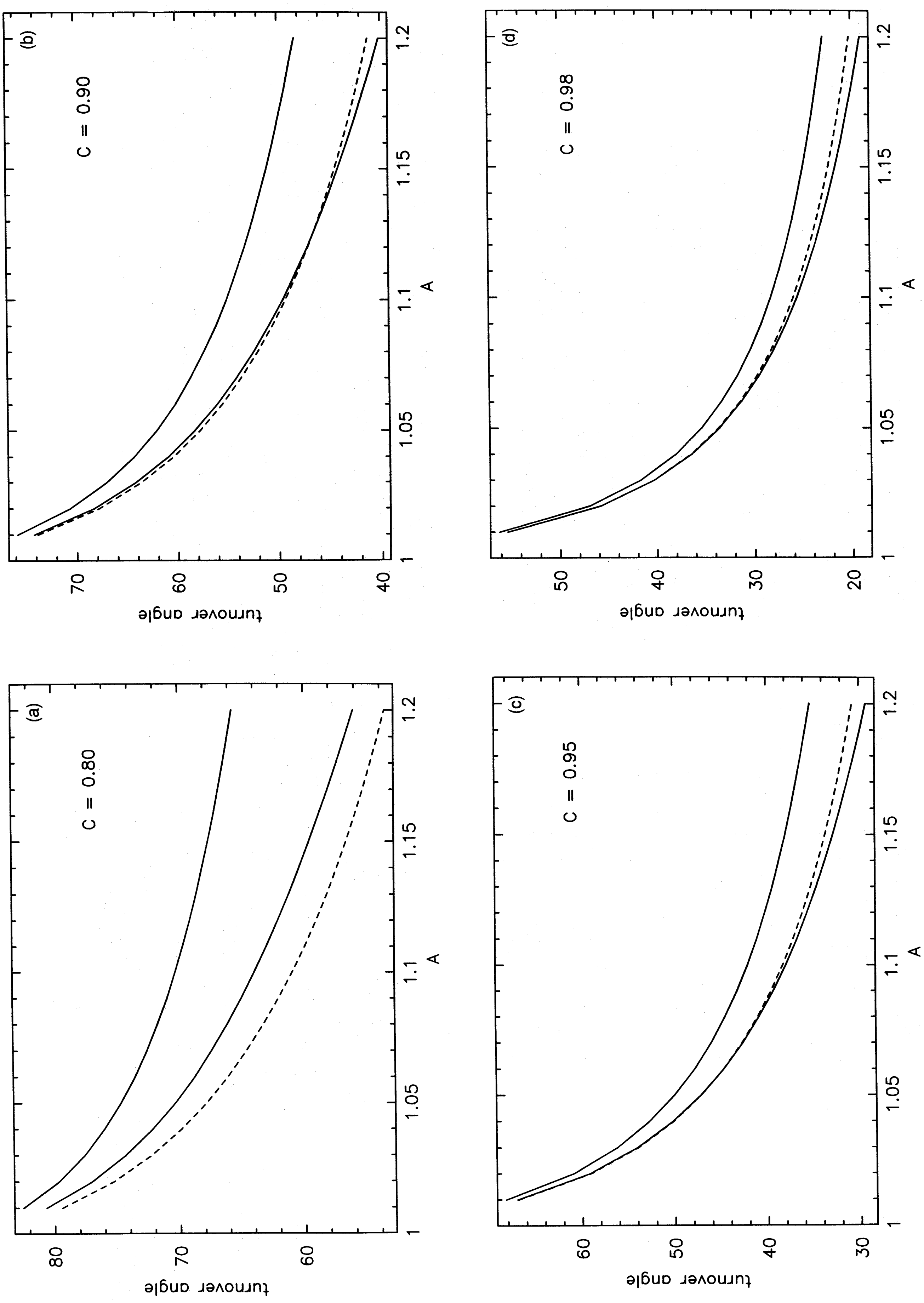

Figure 4. Maximum and minimum transition angles (solid lines), and the theoretical value (dashed line) for a range of triaxialities, as indicated.

Note that this expression is different from that for a system with ellipsoidal density contours, $\rho=\rho[(x/a)^2+(y/b)^2+(z/c)^2]$, for which

$$i_T = \arcsin\left(\frac{b^2-c^2}{a^2-c^2}\right)^{1/2}.$$

These two formulae agree for near-spherical systems, but can differ by 10° or more for moderate triaxialities. For example, $a=1.2$, $b=1$, $c=0.8$ gives $i_T=42.1°$, whereas $A=1.2$, $B=1$, $C=0.8$ gives $i_T=53.6°$.

Particles are given an initial tangential velocity of $\sqrt{2}$, which would lead to a circular orbit in a spherical potential, $A=B=C$. The subsequent behaviour depends both upon the initial orientation of the plane of the orbit and on the orbital phase of the particle. These are labelled by the coordinates shown in Fig. 1. i is the inclination of the orbit to the x–y plane (also the angle between the angular momentum vector, $\boldsymbol{J}$, and the z-axis), Ω is the longitude of the ascending node, and χ is the phase measured from the position of the ascending node. Because the radial velocity is initially zero, χ corresponds also to the phase of the perigee or apogee of the orbit. Note, however, that this phase is not conserved, as it would be in a Keplerian potential.

There are too many free parameters to be able to investigate them all in depth, so we choose to restrict $\boldsymbol{J}$ to lie initially in the x–z plane (i.e. $\Omega=\pi/2$). If the transition lines separating the two regions of precession about the major and minor axes are great circles (as in linear perturbation theory), then it is trivial to generalize our results to arbitrary $\boldsymbol{J}$.

2.2 Results

The measured transition angle, i_S, is a function of the initial phase, χ, as illustrated in Fig. 3 for one particular choice of axis ratios, $A=1.1$, $B=1.0$ and $C=0.9$. The minimum value of i_S occurs at $\chi=0$, and the maximum at $\chi=\pi/2$, with an approximately sinusoidal variation between the two. Hence we need only present results for these two extremes, and the maximum and minimum transition angles for a range of triaxialities are given in Fig. 4. The effect in each case is to reduce the area of the shaded region in Fig. 2, and to smear out the boundary over a few degrees.

As an aside, we never see orbits that switch from precessing about the major to the minor axis, or vice versa. In general, the phase of the apogee of the orbit is not conserved. However, for those orbits that lie close to the transition angle, i_S, the phase returns almost exactly to its original value after one whole precession time. We do not know why this is the case, although it is presumably a reflection of some underlying conservation law.

3 CONCLUSIONS

In this paper, we calculate the simulated transition angle for precession about the major or minor axis of a triaxial potential, and reach the following two conclusions: (i) i_S is almost always greater than i_T, and (ii) i_S is spread out over a few degrees, depending upon the initial phase of the orbit.

The first of these results means that theoretical estimates, based on i_T, of the expected fraction of accreted gas disc that will settle down to give polar rings will be too high. However, the error is not likely to be greater than about 10 per cent, much lower than the uncertainty in the observed frequency of polar ring systems.

More interesting is the second result, which may provide a mechanism for overcoming the angular momentum barrier that prevents accretion into the core of a galaxy. If material is accreted at an inclination between the measured maximum and minimum values of i_S, and if the accreted material is spread out over a range of phases, then precession will occur around both the minor and the major axes. Interactions between the two components can then lead to a large reduction in angular momentum. Indeed, once each component has precessed through 180°, the planes of their orbits again coincide, but their angular momenta are oppositely aligned. Collisions between gas clouds of similar mass may then reduce their velocity relative to the Galactic Centre to almost zero, and they will be accreted into the core. A similar mechanism has been discussed by Steiman-Cameron & Durisen (1984). The twin requirements of a suitable orbital inclination and a large spread in initial orbital phase may not be as unlikely as they at first appear, because the appropriate range of i_S can span up to 10°, even for moderate triaxialities. Also, accretion of a gas-rich dwarf galaxy is likely to populate a fair region of phase space, and a large amount of dissipation is required for this to settle into a disc of gas clouds on circular orbits, as is often assumed for simplicity. We therefore suggest this as one mechanism for fuelling core activity in galaxies.

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