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A lower limit to the binding mass of early-type galaxies

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Summary. We derive a lower limit to the gravitational binding mass of a galaxy in which gas at some high temperature T_0 and pressure P_0 is observed at radius r_0

$$M_{\mathrm{T}} \geqslant \frac{5kT_0 r_0[1-(P_\infty/P_0)^{0.4}]}{2G\mu m_{\mathrm{H}}(1-r_0/r_\infty)}.$$

It is assumed that the gas is confined by a hydrostatic outer atmosphere that is convectively stable and extends to r_∞ where the pressure is P_∞. When applied to the early-type galaxies studied in X-rays by Forman, Jones & Tucker and by Trinchieri, Fabbiano & Canizares, we deduce total masses of between 2 and $12\times10^{12}\,M_\odot$. The average mass-to-light ratio of the galaxies exceeds 74 and so dark matter must be present. If this dark matter is distributed as an isothermal halo then it is 'hotter' than the visible stars in the inner regions of the galaxies.

1 Introduction

X-ray observations of early-type galaxies have shown that they contain large quantities of hot gas (Forman *et al.* 1979; Biermann & Kronberg 1982; Nulsen, Stewart & Fabian 1984; Forman, Jones & Tucker 1985; Trinchieri & Fabbiano 1985). This has raised the possibility (Forman *et al.* 1985) that their total mass distribution, $M(r)$, may be measured using the equation of hydrostatic equilibrium

$$\frac{dP}{dr} = -\varrho\,\frac{d\phi}{dr}, \tag{1}$$

where P and ϱ are the gas pressure and density respectively, and ϕ is the gravitational potential

$$\phi(r) = -\int_r^\infty \frac{GM(r)\,dr}{r^2}. \tag{2}$$

It is likely that the gas is close to hydrostatic equilibrium since otherwise impossibly large mass-flow rates would be implied (Nulsen *et al.* 1984). P and ϱ can be inferred from the X-ray surface brightness profile if the gas temperature, T, is known. As the emission is generated by two-body processes (bremsstrahlung and line radiation), the density profile so obtained is fairly robust, but the temperature profile remains uncertain. This is unfortunate, since (1) may be written as

$$\frac{d\phi}{dr}=-\left(\frac{kT}{\mu m_{\mathrm{H}}}\right)\left(\frac{d\ln\varrho}{dr}+\frac{d\ln T}{dr}\right) \tag{3}$$

and the determination of $\phi(r)$ and thus $M(r)$ depends upon the temperature gradient. Nevertheless, Forman *et al.* (1985) have made estimates of the total masses within the observed radii of their galaxies (~50 kpc) and obtained values of up to $\sim 5\times10^{12}\,M_{\odot}$. These are sufficiently high that they indicate that early-type galaxies possess extensive massive haloes. A detailed study of one of these galaxies, NGC 4472, by Thomas (1986) corroborates this. The uncertainties in this approach have been discussed by Canizares (1986) and Trinchieri, Fabbiano & Canizares (1986). They find that the mass *within the observed region* could in fact be as low as that derived from optical measurements.

In this paper we make use of the limit imposed on the temperature profile by convective stability to establish minimum binding masses of early-type galaxies. These total masses are almost twice those estimated by Forman *et al.* (1985), confirming that the galaxies are surrounded by massive haloes and suggesting that the velocity dispersion in the exterior haloes exceeds that of the visible stars in the inner parts of the galaxy.

2 Mass limits

We are concerned here with minimizing the total binding mass of a galaxy subject to the constraints that (i) gas is observed at some radius r_0 with pressure P_0 and temperature T_0, (ii) the gas distribution is convectively stable, and (iii) the pressure decreases outward. The gas is assumed to be confined by an outer atmosphere extending from r_0 to r_∞ where it terminates or merges with some intergalactic gas at pressure P_∞. We find that the total mass required to bind the observed gas is minimized when the temperature in this outer atmosphere falls as rapidly as possible with radius. The steepest temperature gradient, for which the gas is just convectively stable, corresponds to an adiabat and minimizes the total mass.

In order to show this in a general way, we integrate (1) to give

$$\Delta\phi=\phi(r_\infty)-\phi(r_0)$$

$$=-\int_{P_0}^{P_\infty}\frac{dP}{\varrho},$$

so

$$\Delta\phi=\int_{P_\infty}^{P_0}\frac{kT}{\mu m_{\mathrm{H}}}\frac{dP}{P}. \tag{4}$$

Convective stability means that

$$-\frac{d\ln T}{d\ln r}\leqslant-\frac{(\gamma-1)}{\gamma}\frac{d\ln P}{d\ln r}. \tag{5}$$

Then for a negative pressure gradient, irrespective of the sign of the temperature gradient,

$$T \geqslant \frac{\gamma}{\gamma-1} P \frac{dT}{dP}. \tag{6}$$

We now introduce some arbitrary positive function, $f(P)$, to represent convectively stable departures from an adiabat,

$$T = \frac{\gamma}{\gamma-1} P \frac{dT}{dP} + Pf(P). \tag{7}$$

Then

$$\Delta\phi = \frac{\gamma}{\gamma-1} \frac{k}{\mu m_{\mathrm{H}}} (T_0 - T_\infty) + \int_{P_\infty}^{P_0} \frac{k}{\mu m_{\mathrm{H}}} f(P)\, dP. \tag{8}$$

Multiplying (7) by

$$\left[\frac{(1-\gamma)}{\gamma} P^{(1-2\gamma)/\gamma} \right],$$

and integrating the resultant equation with respect to pressure yields

$$P_\infty^{(1-\gamma)/\gamma} T_\infty - P_0^{(1-\gamma)/\gamma} T_0 = \int_{P_0}^{P_\infty} \frac{(1-\gamma)}{\gamma} P^{(1-\gamma)/\gamma} f(P)\, dP,$$

hence

$$T_\infty = T_0 \left(\frac{P_\infty}{P_0} \right)^{(\gamma-1)/\gamma} + P_\infty^{(\gamma-1)/\gamma} \int_{P_\infty}^{P_0} \frac{(\gamma-1)}{\gamma} \frac{f(P)}{P^{(\gamma-1)/\gamma}}\, dP. \tag{9}$$

Substituting this expression back in (8) gives

$$\Delta\phi = \frac{\gamma}{\gamma-1} \frac{kT_0}{\mu m_{\mathrm{H}}} \left[1 - \left(\frac{P_\infty}{P_0} \right)^{(\gamma-1)/\gamma} \right] + \frac{k}{\mu m_{\mathrm{H}}} \int_{P_\infty}^{P_0} f(P) \left[1 - \left(\frac{P_\infty}{P} \right)^{(\gamma-1)/\gamma} \right] dP. \tag{10}$$

As the second term on the right-hand side of (10) is positive, $\Delta\phi$ is minimized when $f(P)=0$.

Now assuming that $\gamma=5/3$, we obtain

$$\int_{r_0}^{r_\infty} \frac{GM(r)\, dr}{r^2} \geqslant \frac{5}{2} \frac{kT_0}{\mu m_{\mathrm{H}}} \left[1 - \left(\frac{P_\infty}{P_0} \right)^{0.4} \right]. \tag{11}$$

The lowest limit to the total mass is obtained if all the binding mass lies within r_0. Distributing matter beyond r_0 increases the lower limit. To show this, let $M(r_\infty)=M_{\mathrm{T}}$ and $M(r)=M_{\mathrm{T}}-\Delta M$, then

$$\int_{r_0}^{r_\infty} \frac{GM_{\mathrm{T}}\, dr}{r^2} \geqslant \frac{5}{2} \frac{kT_0}{\mu m_{\mathrm{H}}} \left[1 - \left(\frac{P_\infty}{P_0} \right)^{0.4} \right] + \int_{r_0}^{r_\infty} \frac{G\Delta M\, dr}{r^2}, \tag{12}$$

M_{T} is minimized if $\Delta M=0$, thus we finally obtain the lowest limit

$$M_{\mathrm{T}} \geqslant \frac{5}{2} \frac{kT_0 r_0}{G\mu m_{\mathrm{H}}} \frac{[1-(P_\infty/P_0)^{0.4}]}{(1-r_0/r_\infty)}. \tag{13}$$

If the gas were temporarily unstable, convection would set in on a time-scale close to that of free-fall and bring the gas back to a stable situation locally characterized by an adiabat. An identical lower limit to (13) follows from simple escape temperature considerations.

Table 1.

Galaxy (NGC)		T_o(keV)[a]	r_{max}(kpc)[b]	$L_B(10^{10}L_\odot)$	M_T $(10^{12}M_\odot)$	M_{FJT}	M/L_B	σ^c_{lum}(km s^{-1})	r_∞(kpc)
E/S0	315	{1.2}	110 (28)	21.7	12 (3.0)	5.7	55 (14)	273	>1000 (700)
E5	720	1.98	69 (17)	6.8	12 (3.1)	-	181 (45)	224	>>1000 (>>1000)
For A	1316	{1.2}	34 (9)	26.5	3.7 (0.9)	2.0	14 (3)	252	>1000 (370)
S0	1332	{1.0}	36 (9)	4.0	3.2 (0.8)	2.5	81 (20)	306	300 (76)
E2	1395	0.8	74 (19)	5.8	5.4 (1.3)	5.1	92 (23)	249	190 (48)
S0;E	2563	{1.0}	80 (20)	6.8	7.2 (1.8)	4.1	106 (26)	265	>1000 (345)
E1	4374	[1.4]	15 (4)	6.0	1.9 (0.5)	0.86	32 (8)	296	370 (92)
S0 pec	4382	1.3	26 (6)	6.8	3.0 (0.8)	2.5	45 (11)	200	>>1000 (>>1000)
S0/E3[d]	4406	[1.3]	88 (22)	7.3	10.3 (2.6)	4.6	141 (35)	256	>1000 (>1000)
E1/S0[e]	4472	1.2	80 (46)	13.9	8.6 (5)	4.6	62 (36)	315	900 (630)
Sa	4594	[1.6]	32 (8)	20.7	4.6 (1.2)	1.7	22 (6)	256	>1000 (>1000)
E0/S0	4636	1.0	44 (11)	2.7	4.0 (1.0)	2.3	147 (37)	217	>1000 (770)
S0	4649	1.2	36 (9)	8.7	3.9 (1.0)	2.1	45 (11)	344	270 (63)
Cen A	5128	{1.2}	20 (5)	17.1	2.2 (0.5)	1.2	13 (3)	145	>>1000 (>>1000)

a. best-fit values from Trinchieri, Fabbiano & Canizares (TFC;1986) unless [ ] from Forman, Jones & Tucker (FJT; 1985) (average of 90 percent conficence range) or { } estimated from temperature-luminosity distribution obtained from the other galaxies.

b. smaller of values from TFC and FJT and using distances from FJT (H_o = 50 km s^{-1} Mpc^{-1}).

c. σ_{lum} from Whitmore, McElroy & Tonry (1985) and Wilkinson et al. (1986).

d. M86 ram-pressure confined?

e. NGC 4472 is the only galaxy for which the temperature measurements are spatially resolved (Fig.6 of FJT). The bracketed estimate of r_{max} is taken from the outer bin in this figure.

We list in Table 1 the minimum total masses from expression (13) for 14 galaxies well studied in X-rays. The list mainly consists of elliptical and S0 galaxies but includes Fornax A (NGC 1316), the Sombrero galaxy (NGC 4594) and Centaurus A (NGC 5128). We assume that the X-rays originate in diffuse hot gas in all these objects. Some of the emission may be due to X-ray binaries in the lower luminosity galaxies (Trinchieri & Fabbiano 1985), although it should be noted that our limit merely requires the contribution of *some* gas in hydrostatic equilibrium. For the temperature T_0 we adopt either the best-fitting temperature, or the mean of the upper and lower limits to the temperature, found from spectral analysis of the *Einstein Observatory* Imaging Proportional Counter by Trinchieri *et al.* (1986) or Forman *et al.* (1985), respectively. For the radius r_0 we take the smaller of the maximum radii $r_{\max}$ for which X-ray data are reported. P_∞ will be much less than P_0 and is taken to be zero. P_0 typically corresponds to $nT \gg 10^3\,\mathrm{cm}^{-3}\,\mathrm{K}$ in the observed galaxies and is larger than the pressure in the interstellar medium of our galaxy. P_∞ can only approach P_0 for those galaxies in relatively large groups. Even so we may apply (13) to the whole group. As r_0 is then increased by a factor ~10, the mean mass-to-light ratio, M/L_B, of the galaxies in the group is only reduced below that given in Table 1 if there are more than 10 galaxies within that region.

We assume that the measured temperatures represent an average value for each galaxy. It is possible that the temperature at $r_{\max}$ is less than this average and thus that the appropriate value of T_0 for limit (13) is less than that which we have used in Table 1. However, in the two cases where temperature profiles have been reported, NGC 4472 (Forman *et al.* 1985) and M87 (Fabricant & Gorenstein 1983), the measured temperature increases outward. A firm lower limit may be estimated by noting that the $r^{-3/2}$ X-ray surface-brightness profile typical of most early-type galaxies means that half the flux originates within about one quarter of $r_{\max}$. Therefore, since (13) involves the product $T_0 r_0$, we consider that the lowest possible limit is obtained by using $T_0 r_{\max}/4$. Even this limit (given in brackets in Table 1) leads to an average $M/L_B \approx 20$. This is comparable to M/L_B for spirals and indicates that ellipticals possess dark haloes at least as important as those in spiral galaxies. We emphasize that this lowest limit is very conservative and generally implausible since it is only relevant if the temperature has dropped off so fast that it is as low as $T_0/4$ at $r_{\max}$. This implies that much of the luminosity of the gas at the edge of the galaxy falls outside the spectral band of the detector. Simple estimates then show that the total mass of gas there is substantially increased. Furthermore, the short cooling time of the gas in early-type galaxies means that at each radius there is a range of temperatures. Thus we have underestimated the temperature of the hottest gas and consequently T_0. Lastly, we note that low outer temperatures mean very high rates of mass deposition by cooling, far in excess of mass injection by stellar processes.

We emphasize that our mass limit (13) does not minimize the mass within r_0. The mass estimated at a particular radius may be reduced by flattening the local temperature gradient (or even making it positive) but this necessarily increases the total binding mass of the galaxy. This explains why our limits are considerably more than the masses determined from the X-ray profiles at those radii through equation (3) (Forman *et al.* 1985; Trinchieri *et al.* 1986). This disagreement demonstrates that our results hold *a fortiori*. A more reasonable mass distribution requires even greater binding masses (equation 11).

We can proceed further by assuming some form for the distribution of the binding mass. An isothermal halo ($M \propto r$) of line-of-sight velocity dispersion, σ, with a small core radius ($r_{\text{core}} \ll r_0$) gives

$$\sigma^2 = \frac{\phi(r_\infty) - \phi(r_0)}{2\ln(r_\infty/r_0)}. \tag{14}$$

Neglecting the luminous mass (our limits show that $M_{\mathrm{T}}/M_{\text{lum}} \gg 10$, where M_{lum} is the visible mass) then leads to

$$\sigma^2 \geqslant \frac{5}{4}\frac{kT_0}{\mu m_{\mathrm{H}}}\frac{[1-(P_\infty/P_0)^{0.4}]}{\ln(r_\infty/r_0)}. \tag{15}$$

We have used this expression to calculate the outer radius, r_∞, required if the velocity dispersion of the halo is to be as low as that measured optically from the visible stars, σ_{lum}. The results obtained using the measured values of T_0 are listed in Table 1, followed (in brackets) by those obtained using $r_{\mathrm{max}}/4$ for the reasons discussed earlier. In all cases the total binding mass of the isothermal halo is now $\gtrsim 3$ times that obtained from equation (13). As most of these values for r_∞ are unreasonably large, we conclude that the outer massive haloes of elliptical galaxies are hotter than the visible stars in the inner regions.

3 Discussion

We have shown that the average total binding mass associated with early-type galaxies is large, $M_{\mathrm{T}} \gtrsim 5\times10^{12} M_\odot$. This implies that the average mass-to-light ratio $(M/L_B)>74$ and therefore that significant amounts of dark matter are present in early-type galaxies. Our results rely only on the assumption that the gas is in hydrostatic equilibrium. This is reasonable since gas at 10^7 K can travel several Mpc in a Hubble time.

As we have already discussed, it is possible that there is a significant confining pressure due to intracluster or intragroup gas. The lower mass limits given in Table 1 then apply to the whole group. This does not lead to a very interesting conclusion for a central member of a fairly rich group, such as NGC 4472, but it is difficult to envisage a high outer pressure in the case of a relatively isolated elliptical galaxy such as NGC 1395. Any general intercluster medium presumably has only a small pressure, $nT \ll 10^2\,\mathrm{cm^{-3}\,K}$, or else it would close the Universe. Ram pressure probably contributes substantially to P_∞ in the case of NGC 4406 (M86; Forman *et al.* 1979). The X-ray emission should appear to be asymmetrical where stripping is effective; r_{max} may be significantly reduced but T_0 increased by turbulent heating (Nulsen 1982). If the large gaseous envelope of NGC 4406 was gravitationally bound when it was at the outer part of its orbit in the Virgo cluster then our limit in Table 1 is an underestimate of the mass of that galaxy.

Our results also suggest that the dark haloes in early-type galaxies are hotter than the visible stars. A similar result has been obtained for spiral galaxies (the 'cold-bulge syndrome': Whitmore, Kirshner & Schechter 1979; Whitmore & Kirshner 1981). The distribution of the stars in elliptical galaxies is similar in many ways to those of the bulges of spiral and lenticular galaxies. They have similar density profiles (de Vaucouleurs 1959) and, at a given total optical luminosity, they have similar central velocity dispersions, σ_{lum}, and maximum rotation velocities (Davies *et al.* 1983; Kormendy & Illingworth 1983). The X-ray evidence discussed here suggests that the relationship between dark haloes and the luminous spheroidal components is similar in early- and late-type galaxies. The main difference seems to lie in the presence of a disc and its associated cold gas.

The ratio of halo to bulge velocity dispersions has important consequences for theories of galaxy formation (see e.g. Gott 1977 and Faber 1982) and the statistics of gravitational lensing (see e.g. Turner, Ostriker & Gott 1984). For example, the bending angle of a lens (when modelled as a singular isothermal sphere) varies as σ^2 and the cross-section for lensing varies as σ^4. Turner *et al.* adopt $\sigma_{\mathrm{lum}}=306\,\mathrm{km\,s^{-1}}$ for elliptical galaxies with luminosities near L^*. They, and others, have noted the problem of explaining the large number of lenses with large splitting angles using these assumptions.

It should be noted that there is little non-X-ray information about the properties (or even the existence) of heavy haloes around early-type galaxies. Measurements of the velocity-dispersion profiles from absorption-line spectra indicate that the inner parts of the luminous components are self-gravitating or nearly so (Efstathiou, Ellis & Carter 1982; Jedrzejewski 1985). However, even these measurements seldom extend beyond one effective radius, r_e, and thus never more than about 10 kpc from the centres. The optical data are consistent with the presence of heavy haloes so long as these have core radii somewhat larger than r_e. Other methods for inferring the distributions of dark matter at large radii in elliptical galaxies use the spacings of shells (Quinn 1984) or the circular velocities of 'polar' rings. The only results now available, for the shell galaxy NGC 3923 (Hernquist & Quinn 1986) and for the 'polar' rings of two small S0 galaxies (Schweizer, Whitmore & Rubin 1983; Schechter, Ulrich & Boksenberg 1984) are consistent with the X-ray limits presented here.

Equation (11) can also be applied to clusters of galaxies. Henriksen & Mushotzky (1986) obtain an isothermal gas temperature of $\sim 10^8$ K for the gas in the Coma cluster. Modelling and extrapolating the X-ray surface-brightness profile leads to an estimate of the binding mass within 3 Mpc of $8\times10^{14} M_\odot$ (Henriksen & Mushotzky 1986). Our limit using equation (11) gives

$$M_T \gtrsim 10^{15}\left(\frac{T_0}{10^8\,\mathrm{K}}\right)\left(\frac{r_0}{\mathrm{Mpc}}\right) M_\odot. \tag{16}$$

The *Einstein Observatory* X-ray surface-brightness profile is measured out to between 1 and 2 Mpc, so we conclude that the total binding mass of the Coma cluster is at least twice the mass within 3 Mpc deduced by Henriksen & Mushotzky (1986).

The detection of extended X-ray emission from galaxies, groups and clusters leads through inequality (11) to a powerful limit on the total binding mass. In the cases presented here, the total mass exceeds simple estimates of the binding mass within the observed regions obtained from profile modelling.

We conclude that the total binding masses of the galaxies listed in Table 1 are at least $2\text{–}7\times10^{12} M_\odot$. The actual masses are probably somewhat higher. The average mass-to-light ratio of early-type galaxies exceeds 74.

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